

Classical Roots of Inter-universal Teichmüller Theory

Abstract: New advances in mathematics are often portrayed as the ultimate outcome of a strictly linear march, i.e., as the erection of a towering edifice, floor by floor, building on the advances of the state of the art of the previous generation. On the other hand, some advances in mathematics occur in such a way as to bear little resemblance to nearby generations, while sporting a somewhat striking "atavistic" resemblance to generations of the distant past. The present talk will focus on exposing the fundamental conceptual framework of inter-universal Teichmüller theory as a natural, albeit somewhat novel, outgrowth of mathematics that dates back partly to the 1980's (Faltings' invariance of the height of abelian varieties with respect to isogeny), partly to the 1960's (Grothendieck's theory of crystals), partly to the 1930's (classical complex Teichmüller theory), and partly to the nineteenth century (the Jacobi identity for the theta function on the upper half-plane). Just as it is entirely unrealistic to attempt to understand the notion of a Weil cohomology (such as étale cohomology) without first achieving an adequate level of understanding of the notion of singular cohomology in algebraic topology, it is substantially unrealistic to attempt to appreciate the central ideas of inter-universal Teichmüller theory in the absence of a solid grasp of the common thread — consisting of a certain common underlying logical structure — that permeates the (at first glance) somewhat disparate theories listed above (i.e., invariance of the height by isogeny, crystals, classical complex Teichmüller theory, and the functional equation of the theta function). This common underlying logical structure will form the central theme of the present talk.